Let $f(x)=\sqrt{x^2+7}$. **SCORE: ______ / 8 PTS**

[a] Find $f'(x)$.

(1) $\boxed{\lim_{h\to 0}\frac{\sqrt{(x+h)^2+7}-\sqrt{x^2+7}}{h}}\cdot\frac{\sqrt{(x+h)^2+7}+\sqrt{x^2+7}}{\sqrt{(x+h)^2+7}+\sqrt{x^2+7}}$

$=\lim_{h\to 0}\frac{(x+h)^2+7-(x^2+7)}{h(\sqrt{(x+h)^2+7}+\sqrt{x^2+7})}$

(1) $=\boxed{\lim_{h\to 0}\frac{2xh+h^2}{h(\sqrt{(x+h)^2+7}+\sqrt{x^2+7})}}$ $=\boxed{\lim_{h\to 0}\frac{2x+h}{\sqrt{(x+h)^2+7}+\sqrt{x^2+7}}}$ (1)

$=\frac{2x}{2\sqrt{x^2+7}}=\boxed{\frac{x}{\sqrt{x^2+7}}}$ (1)

[b] The position (in inches) of an object moving in along a line is given by $s(t)=\sqrt{t^2+7}$, where t is the time in seconds. Find the instantaneous velocity of the object at time $t=2$. Give the correct units for your answer.

$s'(2)=\frac{2}{\sqrt{2^2+7}}=\boxed{\frac{2}{\sqrt{11}}}$ (1) $\boxed{\text{INCHES/SECOND}}$ (1)

[c] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x=3$.

$f'(3)=\frac{3}{\sqrt{3^2+7}}=\frac{3}{4}$

$\boxed{y-4=\boxed{\frac{3}{4}}(x-3)}$ (1) (1)

Using complete sentences & proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: ______ / 2 PTS**

A FUNCTION f IS CONTINUOUS AT a IF

$f(a)$ EXISTS, $\lim_{x \to a} f(x)$ EXISTS AND $\lim_{x \to a} f(x) = f(a)$

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Determine if the function below is continuous at $x = -1$. **SCORE: ______ / 4 PTS**

State your conclusion clearly, and show whether each condition in the definition of "continuous" is true or false.

In addition, if it is not continuous, determine the type of discontinuity and justify using the appropriate definition.

$$f(x) = \begin{cases} x - x^4 + x^5, & \text{if } x < -1 \\ -1, & \text{if } x = -1 \\ x^7 - x^2 - 1, & \text{if } x > -1 \end{cases}$$

$\frac{1}{2}$ $f(-1) = -1$ EXISTS

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} (x^7 - x^2 - 1) = -1 - 1 - 1 = -3$ $\frac{1}{2}$

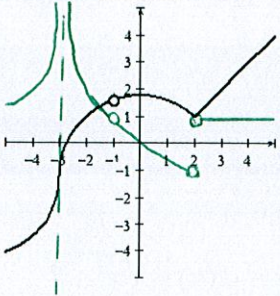
$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} (x - x^4 + x^5) = -1 - 1 - 1 = -3$ $\frac{1}{2}$

$\lim\limits_{x \to -1} f(x) = -3$ EXISTS, $\frac{1}{2}$ BUT $\neq f(-1)$ $\frac{1}{2}$

NOT CONTINUOUS $\frac{1}{2}$

REMOVABLE DISCONT. SINCE $\lim\limits_{x \to -1} f(x)$ EXISTS BUT $\neq f(-1)$

The graph of $f(x)$ is shown on the right. **SCORE: _____ / 6 PTS**



[a] Find all x-coordinates where $f'(x)$ is undefined, and explain **very briefly** why.

1 $x = -3$ VERTICAL T.L.

1 $x = -1$ DISCONT

1 $x = 2$ CUSP

[b] Sketch a graph of $f'(x)$ on the same axes.

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The lifespan of a certain lightbulb depends on how many hours per day it is switched on. **SCORE: ______ / 2 PTS**

Suppose $L = f(u)$, where L is the lifespan of the lightbulb, in months, and u is how many hours per day it is switched on.

[a] What does $f'(5) = -1$ mean ? Your answer must use all the numbers from that equation, and the correct units for those numbers.

IF A LIGHTBULB IS SWITCHED ON 5 HOURS A DAY, ITS LIFESPAN IS SHORTENED BY 1 MONTH FOR EACH ADDITIONAL HOUR IT IS ON EACH DAY

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[b] Is there a value u_0 such that $f'(u_0) > 0$? Why or why not ?

NO. LEAVING THE LIGHTBULB ON LONGER EACH DAY WILL ALWAYS CAUSE IT TO HAVE A SHORTER LIFESPAN

Using complete sentences & proper mathematical notation, write the formal definition of "derivative (function)". **SCORE: ______ / 1 PT**

THE DERIVATIVE OF A FUNCTION f IS $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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Find the following limits.
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

SCORE: ______ / 7 PTS

[a] $\lim_{x\to\infty} \frac{3}{1-4\tan^{-1}x}$

$= \frac{3}{1-4(\frac{\pi}{2})}$

$= \boxed{\frac{3}{1-2\pi}}$ 1

[b] $\lim_{x\to-\infty} \frac{\sqrt{7x^{10}-x}}{13x^3+2x^5} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}}$

$= \lim_{x\to-\infty} \frac{\sqrt{7x^{10}-x}}{13x^3+2x^5} \cdot \boxed{\frac{-\sqrt{\frac{1}{x^{10}}}}{\frac{1}{x^5}}}$ 1 $\frac{1}{2}$

$= \boxed{\lim_{x\to-\infty} \frac{-\sqrt{7-\frac{1}{x^9}}}{\frac{13}{x^2}+2}}$ 1

$= -\frac{\sqrt{7-0}}{0+2}$

$= \underbrace{-}_{\frac{1}{2}}\frac{\sqrt{7}}{\underbrace{2}}$ 1

[c] $\lim_{x\to 0^+} \frac{2-x}{1-e^x}$ $\qquad \frac{2}{0^-}$

$= \underbrace{-}_{1}\underbrace{\infty}_{1}$